नोट : प्रथम, द्वितीय, तृतीय एवं चौथा प्रश्नपत्र हल करना अनिवार्य है। वैकल्पिक ग्रुप में से कोई 01 प्रश्नपत्र हल करना अनिवार्य है। कुल 05 प्रश्नपत्र हल करना अनिवार्य है।

Note: *Each section is compulsorily written on separate answer sheet.*

H-2541

M. A. / M. Sc. (Second Semester) Examination, 2021

MATHEMATICS

Paper : First

(Advanced Abstract Algebra-II)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *Attempt all questions. Each question carry equal marks. Each question must be answered in maximum 800 words.*

1. If A and B are submodules of an R-module M, then prove that the sum $A+B$ is also a submodule of M.

2. Prove that $\text{Hom}_R(M,M)$ is a skew-field where M is a simple module.

3. State and prove Hilbert basis theorem.

4. Let M be a notherian module or any sub-module over a noetherian ring. Then prove that each non-zero submodule contains a uniform module.

5. If $T \in A(V)$ is nilpotent, then prove that

$$\alpha_0 + \alpha_1 T + \ldots\ldots + \alpha_m T^m$$

where $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.

H-2542

M. A. / M. Sc. (Second Semester) Examination, 2021

Paper : Second

(Lebesgue Measure & Integration)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *Attempt all questions. Each questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Prove that every interval is measurable.

2. State and prove Lebesgue Monotone convergence theorem.

3. Let $[a,b]$ be a finite interval and let $f \in L(a,b)$ with indefinite integral F, then prove that $F' = f$ *a.e.* in $[a,b]$.

4. State and prove Minkowski's inequality.

5. Explain almost uniform convergence and show that if $f_n \to f$ a.u., then $f_n \to f$ a.e.

H-2543

M. A. / M. Sc. (Second Semester) Examination, 2021

Paper : Tjord

(Topology-II)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. Each questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Prove that every completely regular space is regular.
2. Let X be a Hausdorff space. Then prove that X is locally compact at x if and only if for every neighbourhood U of x such that $\bar{V}$ is compact and $\bar{V} \subset U$.
3. Prove that the product of finitely many connected spaces is also connected.
4. Let X, Y be topological spaces, $x_0 \in X$ and $f : X \to Y$ a function. Then prove that f is continuous at x_0 if and only if whenver a net S converges to x_0 in X, the net $f \circ S$ converges to $f(x_0)$ in Y.
5. Define path connected space and simply connected space. Prove that in a simply connected space X, any two paths having the same initial and final points are path homotopic.

H-2544

M. A. / M. Sc. (Second Semester) Examination, 2021

Paper : Fourth

(Complex Analysis-II)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Prove the relation between Gamma and Zeta function.

2. State and prove Runge's theorem.

3. State and prove Monodromy theorem.

4. State and prove Jenson's Inequality.

5. Prove that an entire function with more than are finite Lacunary values reduces to a constant.

H-2551

M. A. / M. Sc. (Second Semester) Examination, 2021

Paper : Fifth (i) (Optional)

(Differential Equations-II)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Let $f(t, y, z, z^*)$ be a continuous function on an open (t, y, z, z^*)-set E such that f has continuous partial derivatives of all orders not exceedings m, $m \geq 1$, with respect to the components of y and z. Then

$$y' = f(t, y, z, z^*), \quad y(t_0) = y_0$$

has a unique solution $\eta = \eta(t, t_0, y_0, z, z^*)$ for fixed z, z^* with $(t_0, y_0, z, z^*) \in E$, and η has all continuous partial derivatives of the form

$$\frac{\partial^{i+i_0+\alpha_1+\ldots+\alpha_d+\beta_1+\ldots+\beta_e}\eta}{\partial t^i \partial t_0^{i_0} \partial(y_0^1)^{\alpha_1} \ldots \partial(y_0^d)^{\alpha_d} \partial(z')^{\beta_1} \ldots \partial(z^e)^{\beta_e}},$$

where $i \leq 1$, $i_0 \leq 1$ and $i_0 + \Sigma\beta_k + \Sigma\alpha_j \leq m$.

2. Assume that $f(y)$ is continuous on an open y-set E and that $C^+ : y = y_+(t)$ is a solution of $y' = f(y)$ for $t \geq 0$. Then $\Omega(C^+)$ is closed. If C^+ has a compact closure in E, then $\Omega(C^+)$ is connected.

3. Let $q(t)$ be real valued and continuous for $J : a \leq t < w (\leq \infty)$. Then $u'' + q(t)u = 0$ is disconjugate on J if and only if there exists a continuously differentiable function $r(t)$ for $a < t < w$ such that $r^1 + r^2 + q(t) \leq 0$.

4. Let $A(t)$ be continuous and of period p. Then for a fixed continuous $g(t)$ of period p,

$$y' = A(t)y + g(t)$$

has a solution of period p if and only if

$$y' = A(t)y + g(t)$$

has atleast one bounded solution for $t \geq 0$.

5. Let $f(t, x, x')$ be continuous for $(0 \le t \le p)$ and all (x, x') and satisfy a Lipschitz condition with respect to x, x' of the form

$$\| f(t, x_1, x_1') - f(t, x_2, x_2') \| \le \theta_0 \| x_1 - x_2 \| + \theta_1 \| x_1' - x_2' \|$$

with Lipschitz constants θ_0, θ_1 so small that

$$\frac{\theta_0 p^2}{8} + \frac{\theta_1 p}{2} < 1$$

Then $x'' = f(t, x, x')$ has a unique solution satisfying $x(0) = 0$, $x(p) = 0$.

H-2552

M. A. / M. Sc. (Second Semester) Examination, 2021

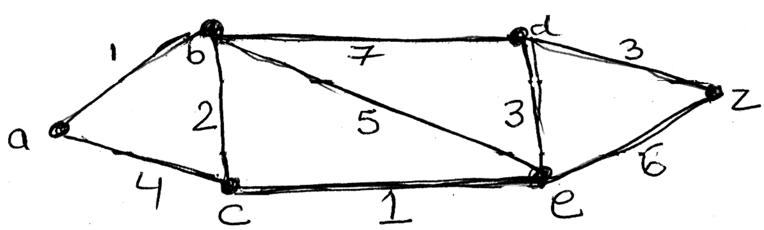
Paper : Fifth (ii) (Optional)

(Advanced Discrete Mathematics-II)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Using Dijkstra's Algorithm find the shortest path from a to z in the given graph where numbers associated with the edges are the weights.



2. Draw and describe the state graph for the finite state machine given below :

State	Input			Output
	0	1	2	
S_0	S_0	S_1	S_2	0
S_1	S_1	S_2	S_0	1
S_2	S_2	S_0	S_1	2

3. Consider the Moore machine described by the given table. Construct the corresponding Mealy machine.

Present State	Next State		Output
	$a = 0$	$a = 1$	
S_1	S_1	S_2	0
S_2	S_1	S_3	0
S_3	S_1	S_3	1

4. Define turning machine and give an example.
5. State and prove pumping lemma.

H-2553

M. A. / M. Sc. (Second Semester) Examination, 2021

MATHEMATICS

Paper : Fifth (iii) (Optional)

(Differential Geometry of Manifolds-II)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Describe in detail with example the induced bundle.
2. State & prove Schur's lemma.
3. If $y:[a, b] \to m$ be a continuous curve with finite length in a Riemannian manifold, then show that y can be uniformly approximated by broken geodesic.
4. Describe the Mainardi-Codazzi equations.
5. Describe contravariant and covariant almost analytic vector fields.

H-2554

M. A. / M. Sc. (Second Semester) Examination, 2021

MATHEMATICS

Paper : Fifth (iv) (Optional)

(Programming in 'C')

Maximum Marks : 25 (Regular) / 35 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Write a C-program to check whether a given year is a leap year or not.
2. Explain Break Statement with suitable program.
3. Explain increment and decrement operators with example. If $x = 4, y = 3$ then find $z = (x++) + (y++)$.
4. Describe the procedure of passing an array in the function argument.
5. Write a C-program to implement structure of an employee that cotain fields emp ID, emp_Name, emp-designation and emp_salary and submission of N employee salary.